Liquid Hydrocarbon Measurement
Uncertainty Calculations

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Introduction

This document provides a methodology for calculating the uncertainty in the measurement of liquid hydrocarbons by flow measurement systems. Specifically, uncertainty performance expressions are developed for positive displacement and Coriolis meter types under API MPMS Chapter 5 for measuring hydrocarbons by volume. This includes ancillary devices, processes, and measurements used to calculate a net standard volume (NSV) such as pressure, temperature, density, and sediment and water. This document was written to determine if the performance of a given measurement device is in compliance with an acceptable level of uncertainty, either pre-installation, or after the installation is in service. If this is not the case, the measurement devices, its installation, or maintenance practices, etc. can be upgraded to meet the desired uncertainty performance criterion. Techniques are described to assess the uncertainty contribution of individual components of measurement systems and the overall measurement facility’s measurement uncertainty. By following the guidance and calculation procedures of this document, cost effective measurements of appropriate quality can be achieved. In most cases the rigorous requirements of industry standards intended for allocation and custody transfer quality measurements can be reduced and still achieve the desired measurement uncertainty required. For this document, a measurement system is comprised of meters, provers, and the associated devices to calculate a net standard volume (NSV).

This document addresses the most common liquid measurement devices in use at the time of its development and allows for updating for new devices in the future. This does not advocate the use of these devices or preclude the utilization of other types of devices, provided the targeted performance is achieved.
Liquid Hydrocarbon Measurement Uncertainty Calculations

1 Scope

This document provides guidelines for the calculation of uncertainty for field stored and transported hydrocarbon liquids. Special emphasis is placed on the measurement uncertainty of crude oils measured at tanks, by lease automatic custody transfer (LACT), and alternative measurement systems.

2 Terms and Definitions

For the purposes of this document, the following definitions apply.

2.1 accuracy
The closeness of agreement between a measured quantity value and a true quantity value of a measurand.

2.3 calibration
A set of operations which establish, under specified conditions, the relationship between the values indicated by a measuring device and the corresponding known values indicated when using a suitable measuring standard.

2.12 metering or measurement system
A combination of primary, secondary and/or tertiary measurement components necessary to determine the flow rate.

2.14 performance
The response of a measurement device to influence parameters such as operating conditions, installation effects, and fluid properties.

2.16 uncertainty
The range or interval within which the true value is expected to lie with a stated degree of confidence. Describes the range of deviation between a measured value and the true value, expressed as a percentage. For example, a device with an accuracy of 2% would have an uncertainty of ±2%.

3 Performance Characteristics and Measurement by Meter Type

3.1 General

The primary purpose of a liquid hydrocarbon meter for any application is to measure the flow. The uncertainty of measurement depends on the measurement equipment selected for the application, proper installation of the equipment, the ability to inspect, verify, or calibrate the various measurement system components, and the frequency of those maintenance activities. The performance of the meter may also depend on the piping configuration and compensation for variability of operating pressure, temperature, and fluid composition. It is important to recognize individual influence parameters and their effect on the measurement. Since the principle of operation and differing influence parameters have varying degrees of influence by meter type, it is important to identify and define the significant influence factors for the meter to determine the total or combined measurement uncertainty.
3.2 Uncertainty Requirements

3.2.1 General

The purpose of this document is to develop the uncertainty representations and example calculations for meters, provers, and associated equipment used on any metering system such as LACT systems which can be used to address uncertainty requirements. Focus is placed on Coriolis and positive displacement meters since they are the most commonly used on LACT systems to measure crude oil quantities. With the proper understanding of individual system component uncertainties, the overall system uncertainty can be calculated for metering systems in the design phase, or already in operation in the field.

3.2.2 Specifics of Requirements

The sources of uncertainty that are addressed arise from:

- Meter measurement
  - Linearity;
  - Thermal and pressure variation;
  - Deviation from meter factor proving conditions.

- Fluid properties
  - Thermal, pressure, and viscosity variation;
  - Profile deviation impact.

- Prover
  - Temperature, pressure variation.

- Associated devices
  - S&W, density.

- Calibration
  - Temperature, pressure, density, viscosity, S&W.

4 Uncertainty Development

4.1 General

The work by Dahl, et al (2003) was used as basis for the development of this document. Refer to Annex A for examples of the uncertainty calculation procedure.

4.2 Basic Equations

4.2.1 General


The following equations can be used to represent error in flow:

\[ q = NSV = GSV \ast CSW \]
where:

$q$ is the net standard volumetric flow rate,

GSV is the gross standard volumetric flow rate,

CSW is the correction for sediment and water.

\[
CSW = 1 - \frac{\%SW}{100}
\]

Using the basic equations developed in the handbook by Dahl, et al (2003):

\[
\left( \frac{\delta q_{v0,me}^2}{q_{v0,me}} \right)^2 = \left( \frac{\delta (A_{liq} m_{\Delta p, C})}{A_{liq} A_{steel}} \right)^2 + \left( \frac{\delta (q_{v0}^\text{cal})}{q_{v0}^\text{cal}} \right)^2 + \left( \frac{\delta (q_{v0}^\text{prov})}{q_{v0}^\text{prov}} \right)^2 + \left( \frac{\delta (q_{v0}^\text{met})}{q_{v0}^\text{met}} \right)^2 + \left( \frac{\delta (q_{v0}^\text{SW})}{q_{v0}^\text{SW}} \right)^2 + \left( \frac{\delta (q_{v0}^\text{flow})}{q_{v0}^\text{flow}} \right)^2
\]

where $v_0$ is intended to signify the measurement by the primary flow meter after corrections from the proving and calibrations have been applied. Each term of this development is discussed below.

4.2.2 Overall System Uncertainty

The term \( \left( \frac{\delta q_{v0,me}^2}{q_{v0,me}} \right)^2 \) on the left-hand side of Eq. 1 represents the square of the overall uncertainty of the metering system in percent. The uncertainty is typically calculated as the square root of the sum of the squares of the individual uncertainties from the other quantities of Eq. 1:

\[
\frac{\delta q_{v0,me}}{q_{v0,me}} = \sqrt{\left( \frac{\delta (A_{liq} m_{\Delta p, C})}{A_{liq} A_{steel}} \right)^2 + \left( \frac{\delta (q_{v0}^\text{cal})}{q_{v0}^\text{cal}} \right)^2 + \left( \frac{\delta (q_{v0}^\text{prov})}{q_{v0}^\text{prov}} \right)^2 + \left( \frac{\delta (q_{v0}^\text{met})}{q_{v0}^\text{met}} \right)^2 + \left( \frac{\delta (q_{v0}^\text{SW})}{q_{v0}^\text{SW}} \right)^2 + \left( \frac{\delta (q_{v0}^\text{flow})}{q_{v0}^\text{flow}} \right)^2}
\]

4.2.3 Oil and Steel Expansion Uncertainty Contribution

The first term to the right of the equal ('=') sign in Eq. 1 represents the expansion or contraction of the liquid and of the steel in the measurement system. This includes hydrocarbon flowing in the meter, prover, and calibrated devices due to variations in temperature, pressure, and density.

\[
\left( \frac{\delta (A_{liq} m_{\Delta p, C})}{A_{liq} A_{steel}} \right)^2
\]

where:

\[
A_{liq} = \left( \frac{C^\text{prov}_{t tp} C^\text{prov}_{plp} C^\text{met}_{tim} C^\text{met}_{plm}}{C^\text{prov}_{t im} C^\text{prov}_{plm}} \right)
\]
The oil and steel expansion term is a function of $T_p^{\text{cal}}$, $p_p^{\text{cal}}$, $T_p^{\text{prov}}$, $p_p^{\text{prov}}$, $T_m^{\text{prov}}$, $p_m^{\text{prov}}$, $T_m^{\text{met}}$, $p_m^{\text{met}}$, and $\rho_o$ where $T_p^{\text{cal}}$ and $T_p^{\text{prov}}$ are the temperatures at the prover during 1) calibration and 2) proving, as measured by the same temperature device. $p_p^{\text{cal}}$ and $p_p^{\text{prov}}$ are the same with pressure. Note that $T_p^{\text{cal}}$ and $p_p^{\text{cal}}$ are uncorrelated with $T_p^{\text{prov}}$ and $p_p^{\text{prov}}$. $T_m^{\text{prov}}$ and $T_m^{\text{met}}$ are measured by the same temperature device, as are $p_m^{\text{prov}}$ and $p_m^{\text{met}}$ and therefore these measurements sets are correlated.

### 4.2.4 Calibration Uncertainty Contribution

Uncertainty contributions from the calibration of the devices in the measurement system is composed of the uncertainty in the reference devices used for calibration, and the error in the calibration itself.

$$\left( \frac{\delta(q_{v_o}^{\text{cal}})}{q_{v_o}^{\text{cal}}} \right)^2 = \left( \frac{\delta(q_{v_o,\text{ref}}^{\text{cal}})}{q_{v_o}^{\text{cal}}} \right)^2 + \left( \frac{\delta(q_{v_o,\text{prov}}^{\text{cal}})}{q_{v_o}^{\text{cal}}} \right)^2$$

Here the uncertainty term $\delta(q_{v_o,\text{prov}}^{\text{cal}})$ refers to the on-site prover system at on-site calibration or water draw. Other devices, temperature, pressure, and density have terms that are expressed in the quantity $\delta(q_{v_o,\text{ref}}^{\text{cal}})$.

### 4.2.5 Prover Uncertainty Contribution

This term is uncertainty due to the proving process (not caused by calibration). It applies to single flow proving only.

$$\left( \frac{\delta(q_{v_o}^{\text{prov}})}{q_{v_o}^{\text{prov}}} \right)^2 = \left( \frac{\delta(q_{v_o,\text{prover}}^{\text{prov}})}{q_{v_o}^{\text{prov}}} \right)^2 + \left( \frac{\delta(q_{v_o,\text{flowmeter}}^{\text{prov}})}{q_{v_o}^{\text{prov}}} \right)^2 + \left( \frac{\delta(q_{v_o,\text{linearity}}^{\text{prov}})}{q_{v_o}^{\text{prov}}} \right)^2 + \left( \frac{\delta(q_{v_o,\text{profile}}^{\text{prov}})}{q_{v_o}^{\text{prov}}} \right)^2$$

The uncertainty impact of medium temperatures and pressures, and of the calibration temperatures and pressures was captured in section 4.2.2 and 4.2.3. This uncertainty refers to the repeatability of the prover, the repeatability of the meter, the uncertainty effect of carrying out the proving at a flowrate different from the calibration flowrate (linearity) but can also include pulse resolution impacts. The last “profile” term is used to catch any other known sources of uncertainty including viscosity and other uncertainty impacts that can be determined from data.

One of the contributions to the proving uncertainty is the flowmeter uncertainty during proving term,

$$\left( \frac{\delta(q_{v_o,\text{flowmeter}}^{\text{prov}})}{q_{v_o}^{\text{prov}}} \right)^2$$

which includes the uncertainty of the indicated gross standard volume total while proving:

$$\left( \frac{\delta(q_{v_o}^{\text{prov}})}{q_{v_o}^{\text{prov}}} \right)^2$$
Where,

\[ q_{v_o}^{IV} = G_{SV} = C_{TL} * C_{PL} * I_V * M_F = M_F * C_{TL} * C_{PL} * \Delta N_{p}^{IV} * K \]

Where \( \Delta N_{p}^{IV} \) is the number of pulses for registration of the meter total output during each proving pass.

K is the “k” factor, denotes BBL/pulses. \( K = f(T, P, \rho, \delta \Delta N_p) \).

In applications where pulse interpolation is not needed (e.g., when pulses per prover pass \( \geq 10,000 \)): \( n_{\text{umber of pulses per prover pass}} \)

\[ \left( \frac{\partial(q_{v_o}^{IV})}{\partial(\Delta N_p)} \right) \delta(\Delta N_p) = \frac{1}{\text{number of pulses per prover pass}} \]

4.2.6 Meter Uncertainty Contribution

The fourth term on the right-hand side of the equal sign (‘\( = \)’) in Eq. 1 is the contribution to the uncertainty by the meter and the process of metering. There are three parts to this term that can be identified:

\[ \left( \frac{\partial(q_{v_o}^{IV})}{\partial(a_{v_o})} \right)^2 = \left( \frac{\partial(q_{v_o}^{IV})(\text{flowmeter})}{\partial(q_{v_o}^{IV})} \right)^2 + \left( \frac{\partial(q_{v_o}^{IV})(\text{linearity})}{\partial(q_{v_o}^{IV})} \right)^2 + \left( \frac{\partial(q_{v_o}^{IV})(\text{profile})}{\partial(q_{v_o}^{IV})} \right)^2 \]

The first of the meter uncertainty terms, \( \left( \frac{\partial(q_{v_o}^{IV})(\text{flowmeter})}{\partial(q_{v_o}^{IV})} \right) \), includes the random uncertainty, and any uncertainty due to pressure, temperature, and fluid density influence effects on the flowmeter’s direct measurement. This term can usually be determined from either vendor data and/or proving data and is usually expressed in terms of percent over the range of flow.

\[ \left( \frac{\partial(q_{v_o}^{IV})(\text{profile})}{\partial(q_{v_o}^{IV})} \right)^2 \]

Where

\[ \left( \frac{\partial(q_{v_o}^{IV})(\text{profile})}{\partial(q_{v_o}^{IV})} \right)^2 = \left( \frac{\partial(q_{v_o}^{IV})(\text{random})}{\partial(q_{v_o}^{IV})} \right)^2 + \left( \frac{\partial(q_{v_o}^{IV})}{\partial(q_{v_o}^{IV})} \right)^2 \]

And where:

\[ \left( \frac{\partial(q_{v_o}^{IV})}{\partial(q_{v_o}^{IV})} \right)^2 = \left( \frac{\partial(q_{v_o}^{IV})}{\partial(T)} \right)^2 \delta(T) + \left( \frac{\partial(q_{v_o}^{IV})}{\partial(P)} \right)^2 \delta(P) + \left( \frac{\partial(q_{v_o}^{IV})}{\partial(\rho)} \right)^2 \delta(\rho) \]
Where \( \left( \frac{\partial (q_{v_0})}{\partial T} \right) \delta (T) \), \( \left( \frac{\partial (q_{v_0})}{\partial P} \right) \delta (P) \), and \( \left( \frac{\partial (a_{v_0})}{\partial \rho} \right) \delta (\rho) \) are the uncertainty terms of the meter’s indicated value as affected by influence factors of temperature, pressure, and fluid density, respectively.

The second meter uncertainty term, \( \left( \frac{\delta (q_{v_0, \text{linearity}})}{q_{v_0, \text{met}}} \right) \), includes the uncertainty due to linearity of the flow meter when operation is at a flow rate close to where the meter was proved. It is based on the specification of the total linearity of the meter over the calibrated range of volumetric flow rate at standard conditions. It is given as a linearity percent of \( L \) where this is represented as linear drift of \( L\% \) over the calibrated range (the flow rate) of the meter. This term can also include other previously undetermined flowrate uncertainties.

When proved at \( q_{v_0, \text{prov}} \) and operating at \( q_{v_0, \text{met}} \), the linearity uncertainty contribution is:

\[
\left( \frac{\delta (q_{v_0, \text{linearity}})}{q_{v_0, \text{met}}} \right)^2 = \frac{L}{\sqrt{3}} \frac{q_{v_0, \text{met}} - q_{v_0, \text{prov}}}{q_{v_0, \text{cal}} - q_{v_0, \text{cal}}} 
\]

This is known as rectangular uncertainty.

The third meter uncertainty term, \( \left( \frac{\delta (q_{v_0, \text{profile}})}{q_{v_0, \text{met}}} \right) \), includes the uncertainty due to any effects on the meter measurement caused by variations in the profile of the flow stream. Positive displacement meters and Coriolis meters are unaffected by profile variations, but other meter types (e.g., orifice, turbine, and ultrasonic meters) may have uncertainty terms related to profile variations that will depend on the straight-run lengths and flow conditioning measures included in the meter run installation design. Manufacturers should be consulted for profile-related uncertainty values.

The fourth meter uncertainty term, \( \left( \frac{\delta (q_{v_0, \text{drift}})}{q_{v_0, \text{met}}} \right) \), includes the uncertainty due to changes in a meter over time between proving that are consistently in one direction due to wear, coating, or other cumulative effects. Drift effects can vary depending on meter type and process fluid properties and composition. Manufacturers should be consulted for drift-related uncertainty values.

Pulse stability (Ps), as defined in API MPMS Chapter 4.2 \(^{[1]}\), can also contribute to both the meter (met) uncertainty term and the proving (prov) uncertainty term. The pulse stability can become a more significant factor to the proving uncertainty results, especially when the prover size is small enough to warrant pulse interpolation. Because the contribution of pulse stability to the uncertainty of a batch measurement diminishes as the batch size increases, the contribution of pulse stability to the meter uncertainty term in normal operation is generally insignificant compared to the contribution of pulse stability to the proving uncertainty term.

### 4.2.7 Sediment and Water Determination Uncertainty Contribution

The fifth term on the right-hand side of the equal sign (‘=’) in Eq. 1 is the contribution to the uncertainty from the process of determining and adjusting for sediment and water in the oil stream. This term is:
And from the definition on the sediment and water correction factor (see API MPMS Chapter 12.2.1 [5]),

$$CSW = 1 - \frac{\%SW}{100\%}$$

Since

$$C_{v_o}^{SW} (T, P) = \frac{q_{v_o, m.c} \times \%SW}{100\%}$$

And

$$SW = f(P, T, l)$$

$$\left( \frac{\delta(q_{v_o}^{SW})}{q_{v_o}^{SW}} \right)^2 = \left[ \frac{1}{100q_{v_o}^{SW}} \times \left( \frac{\partial SW}{\partial T} \right) \right]^2 + \left[ \frac{1}{100q_{v_o}^{SW}} \times \left( \frac{\partial SW}{\partial P} \right) \right]^2 + \left[ \frac{1}{100q_{v_o}^{SW}} \times \left( \frac{\partial SW}{\partial l} \right) \right]^2$$

Additionally,

$$\frac{\partial SW}{\partial T} = \delta SW \times C_{TL}^{SW} (T)$$

$$\frac{\partial SW}{\partial P} = \delta SW \times C_{PL}^{SW} (P)$$

$$\frac{\partial SW}{\partial l} = \delta SW$$

Where $P$ is sample pressure, $T$ is sample temperature, and $l$ is sample size. Using these relations, it can be re-written as:

$$\left( \frac{\delta \left( q_{v_o}^{SW} \right)}{q_{v_o}^{SW}} \right)^2 = \left( \frac{1}{100q_{v_o}^{SW}} \right)^2 \left[ \left( \delta SW \times C_{TL}^{SW} (T) \right)^2 + \left( \delta SW \times C_{PL}^{SW} (P) \right)^2 + (\delta SW)^2 \right]$$

Therefore, the uncertainty can be determined and used in the term that represents sediment and water uncertainty directly.

### 4.2.8 Flow Uncertainty Contribution

The last term in Eq.1 represents the impact of flow conditions not previously captured by those in proving and metering that can cause inaccuracy due to the metering system being used with a meter factor away from the point at which the system was proved.

$$\left( \frac{\delta (q_{v_o}^{flow})}{q_{v_o}^{flow}} \right)^2$$
The uncertainty represented by this term is related to the difference in the meter factors that can be determined over the range of operating conditions for the metering system. It is generally obtained from proving at multiple operating conditions that would represent the largest and smallest meter factors that can be obtained for the metering system. Note that proving at multiple operating conditions and using a mean-average meter factor, scheduling meter factors with flow rate changes, and various forms of linearity compensation will all impact the magnitude of this term.
Annex A
(informative)

Uncertainty Examples

The following examples are merely for illustration purposes only. They are not to be considered exclusive or exhaustive in nature. API makes no warranties, express or implied for reliance on or any omissions from the information contained in this document.

The examples given in this section are based on the equations and processes set forth in Section 4 of this document.

**EXAMPLE 1: Positive Displacement Meter with Bi-Directional Pipe Prover**

![Diagram of LACT Measurement System with PD Meter and Bi-Directional Prover](image)

**Figure A.1—LACT Measurement System with PD Meter and Bi-Directional Prover**
A 6 in. PD meter is nominally flowing at 4000 BPH, at 100 psig and 80 °F. The prover is measured to be 20 BBL. The pipe prover is proved at 70 °F and 100 psig. ρ=30 API. Find the uncertainty of the devices.

**Meter:**

\[ T_{\text{met}} = 80 \, F, \quad \delta T_{\text{met}} = 0.5 \, F \]
\[ P_{\text{met}} = 114.7 \, psig, \quad \delta P_{\text{met}} = 0.5 \, PSIA \]
\[ \rho_{\text{met}} = \frac{141.5}{131.5 + API} = 0.8761 \times 999.012 \, \frac{kg}{m^3} = 875.30 \, \frac{kg}{m^3} \]
\[ \delta API = 0.1 \]
\[ \delta \rho_{\text{met}} = \frac{\delta API}{131.5 + API} \times \rho_{60} \]
\[ = \frac{0.1}{131.5 + 30} \times 875.30 \, \frac{kg}{m^2} \]
\[ \delta \rho_{\text{met}} = 0.54 \, \frac{kg}{m^3} \]
\[ SW = 0.10\%, \quad \delta SW = 0.02\% \]

Meter steel expansion \( \gamma_{\text{met}} = 1.86 \times 10^{-5} \, ^{\circ}F^{-1} \) at 60 °F,
and \( \beta_{\text{met}} = 8.1 \times 10^{-7} \, psig^{-1} \) at 14.7 psia.

Vendor gives linearity of flow meter as 0.05% of full scale between 500 and 6000 BPH. Meter factor tracking data shows repeatability to be 0.037% within operating range. Meter K factor is 8400 pulses per BBL. Operation is routinely between 3,600 and 4,800 BPH due to pumping constraints.

The pulse resolution is BBL/8400 pulses which results in a pulse uncertainty of 1.2x10⁻⁴ BBL.

**Calibration:**

\[ T_{\text{cal}} = 70 \, F, \quad \delta T_{\text{cal}} = 0.5 \, F \]
\[ P_{\text{cal}} = 114.7 \, psig, \quad \delta P_{\text{cal}} = 0.5 \, PSIA \]
\[ \rho_{\text{cal}} = 875.30 \, \frac{kg}{m^3}, \quad \delta \rho_{\text{cal}} = 0.54 \, \frac{kg}{m^3} \]

NIST-traceable can provers used to water draw the pipe prover are accurate to 0.005 barrels in 20 barrels, or 0.025% relative accuracy, and repeatability uncertainty of 0.0017%.

**Prover:**

\[ T_{\text{prov}} = 70 \, F, \quad \delta T_{\text{prov}} = 0.5 \, F \]
Prover steel expansion \( \gamma_{prov} = 1.86 \times 10^{-5} \text{°F}^{-1} \) at 60 °F,
and \( IP = 6.00 \text{ inches}, E = 3.0 \times 10^7 \text{psi}, WT = 0.25 \text{ inches} \).

From tracking data, the proving repeatability is 0.015% at the nominal flow rate of 4000 BPH.
Vendor gives linearity for the prover as 0.02% of full scale between 500 and 6000 BPH.

**System Uncertainty**

The terminology used for variables in this example was discussed previously in section 4. From 4.2.2 the system uncertainty can be calculated as:

\[
\frac{\delta q_{v0,me}}{q_{v0,me}} = \pm \sqrt{\left( \frac{\delta \left( A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p, c} \right)}{A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p, c}} \right)^2 + \left( \frac{\delta (q_{cal})}{q_{cal}} \right)^2 + \left( \frac{\delta (q_{prov})}{q_{prov}} \right)^2 + \left( \frac{\delta (q_{met})}{q_{met}} \right)^2 + \left( \frac{\delta (q_{SW})}{q_{SW}} \right)^2 + \left( \frac{\delta (q_{flow})}{q_{flow}} \right)^2}
\]

Each term under the root is discussed and calculated below.

**Oil & Steel Expansion Uncertainty Term**

\[
\left( \frac{\delta \left( A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p, c} \right)}{A_{liq}^{m,\Delta p} A_{steel}^{m,\Delta p, c}} \right)^2
\]

Where:

\[
A_{liq}^{m,\Delta p} = \left( \frac{c_{cal}^{prov} c_{psp}^{prov} c_{tsm}^{prov} c_{psm}^{prov}}{c_{tsm}^{prov} c_{psm}^{prov}} \right),
\]

\[
A_{steel}^{m,\Delta p, c} = \left( \frac{c_{cal}^{prov} c_{psp}^{prov} c_{tsm}^{prov} c_{psm}^{prov}}{c_{tsm}^{prov} c_{psm}^{prov}} \right)
\]

These functions are dependent upon the temperature, pressure, and density such that:

\[
A_{liq}^{\Delta p} A_{steel}^{m,\Delta p, c} = f_2(T_p^{cal}, p_p, T_p^{prov}, p_p^{prov}, T_m^{prov}, p_m^{prov}, T_m^{met}, p_m^{met}, \rho_o)
\]

Given its dependencies, the oil and steel error terms can be written as:
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\[
\left( \frac{\delta(A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m, A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m)}{A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m, A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m, A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m} \right)^2
\]

\[
= \left[ \frac{1}{A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m, A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m, A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m} \right]^2
\]

\[
= \left[ \left( \frac{\delta f}{\partial T_p^{\text{cal}}} \right)^2 + \left( \frac{\delta f}{\partial P_p^{\text{cal}}} \right)^2 \right] \delta(T_p) + \left[ \left( \frac{\delta f}{\partial T_p^{\text{pro}} \partial P_p^{\text{pro}}} \right)^2 \right] \delta(P_p)
\]

\[
\]

If the same measurement device is used for calibration, flow measurement, and proving, that uncertainty is the same. Note that this does not have to be the case. The term "mod" implies that this part is modeled. Thus:

\[
\delta(T_p) = \delta(T_p^{\text{cal}}) = \delta(T_p^{\text{pro}})
\]

\[
\delta(P_p) = \delta(P_p^{\text{cal}}) = \delta(P_p^{\text{pro}})
\]

\[
\delta(T_m) = \delta(T_m^{\text{pro}}) = \delta(T_m^{\text{met}})
\]

\[
\delta(P_m) = \delta(P_m^{\text{pro}}) = \delta(P_m^{\text{met}})
\]

Using these relations, the error term can be rewritten as:

\[
\left( \frac{\delta(A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m, A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m)}{A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m, A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m, A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m} \right)^2
\]

\[
= \left[ \frac{1}{A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m, A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m, A_{\text{liq}}^m A_{\text{steel}}^m A_{\text{c}}^m D_p^m} \right]^2
\]

\[
= \left[ \left( \frac{\delta f}{\partial T_p^{\text{cal}}} \right)^2 + \left( \frac{\delta f}{\partial T_p^{\text{pro}} \partial P_p^{\text{pro}}} \right)^2 \right] \delta(T_p) + \left[ \left( \frac{\delta f}{\partial T_p^{\text{pro}} \partial P_p^{\text{pro}}} \right)^2 \right] \delta(P_p)
\]

\[
+ \left[ \left( \frac{\delta f}{\partial T_m^{\text{pro}} \partial P_m^{\text{pro}}} \right)^2 \right] \delta(T_m) + \left[ \left( \frac{\delta f}{\partial T_m^{\text{pro}} \partial P_m^{\text{pro}}} \right)^2 \right] \delta(P_m)
\]

The liquid portion of the term can be expanded to (eqn A.1):
The general form of the expansion term due to temperature to be used above \( C_{TLx} \) from [6] 11.1.3.3 eqn. 14) can be written as:

\[
C_{TLx} = e^{-\alpha \Delta T_x - 0.8 a^2 \Delta T_x^2}
\]

where: \( \alpha = \frac{k_0}{\rho_0} + \frac{k_1}{\rho_0} + k_2 \) 

(note that \( k_1, k_2 \) are zero for crude oils; \( k_0 = 341.0957 \text{ kg}^2/\text{m}^6 \text{ °F} \)).

\( \Delta T_x = T_x - T_o \)

\( T_o \) is the reference temperature (in this case, 60F).

The general form of the expansion term due to pressure effect \( C_{PLx} \) from [6] 11.1.3.3 eqn. 15), likewise can be written as:

\[
C_{PLx} = \frac{1}{[1 - 100(P_x - P_v)F]}
\]

\( P_v \) is the reference or the vapor pressure of the liquid.

\[
F = e^{A + BT_x + \rho_o^{-2}(C + DT_x)}
\]

From [6] 11.1.3.3, \( A=-1.99470, B=0.00013427, C=793920, D=2326. \)

The steel portion can also be expanded:

\[
\left( \frac{\delta(A_{steel,mod})}{A_{steel}} \right)^2,
\]

where,

\[
A_{steel} = \left( \frac{C_{psp}^{sp} C_{tsp}^{met} C_{psm}^{sp}}{C_{psp}^{sp} C_{tsp}^{met} C_{psm}^{sp}} \right)
\]

Given
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\[ C_{TS_x}^{\text{met}} = 1 + \gamma_{\text{met}}(T_x - T_b) \quad \text{for the meter, and} \]

\[ C_{TS_x}^{\text{cal}} = 1 + \gamma_{\text{prov}}(T_x - T_b) \quad \text{for the prover calibration,} \]

\[ C_{TS_x}^{\text{prov}} = 1 + \gamma_{\text{prov}}(T_x - T_b) \quad \text{for the prover} \]

where \( T_x \) is the temperature of operation and \( T_b \) is the base temperature for the expansion factors \( \gamma_{\text{met}} \) and \( \gamma_{\text{prov}} \). Further, for Coriolis meters and displacement provers:

\[ C_{PS_x}^{\text{met}} = 1 + \beta_{\text{met}}(P_x - P_b) \]

\[ C_{PS_x}^{\text{cal}} = 1 + (P_x - P_b) \left( \frac{IP}{E \cdot WT} \right) \]

\[ C_{PS_x}^{\text{prov}} = 1 + (P_x - P_b) \left( \frac{IP}{E \cdot WT} \right) \]

where \( P_x \) is the operating pressure, \( P_b \) is the base pressure for the expansion factor, \( \beta_{\text{met}} \) is the expansion factor for the meter steel, \( IP \) is the inner diameter of the pipe, \( E \) is the Young's modulus of the pipe, and \( WT \) is the pipe wall thickness (from MPMS Chapter 12.2 [5]).

By chain differentiation:

\[
\left( \frac{\partial (A_{\text{liq}}^{\Delta p,c})}{A_{\text{liq}}^{\Delta p,c}} \right)^2 = \left( \frac{\partial (A_{\text{steel}}^{\Delta p,c})}{\partial \alpha_m} \delta \alpha_m \right)^2 + \left( \frac{\partial (A_{\text{steel}}^{\Delta p,c})}{\partial \beta_m} \delta \beta_m \right)^2
\]

With the numbers presented in the description of the example, the liquid expansion part gives:

\[ C_{\text{tlp}}^{\text{prov}} = 0.99554, \]

\[ C_{\text{plp}}^{\text{prov}} = 1.00053, \]

\[ C_{\text{tim}}^{\text{met}} = 0.99107, \]

\[ C_{\text{plm}}^{\text{met}} = 1.00049, \]

\[ C_{\text{tim}}^{\text{prov}} = 0.99107, \]

\[ C_{\text{plm}}^{\text{prov}} = 1.00049. \]

Therefore,

\[ A_{\text{liq}}^{\Delta p} = 0.99607. \]

And the steel expansion part,

\[ C_{\text{tsp}}^{\text{prov}} = 1.86 \times 10^{-4} \]
\[ C_{p_{sp}}^{prov} = 8.8 \times 10^{-5} \]
\[ C_{tsm}^{met} = 3.72 \times 10^{-4} \]
\[ C_{p_{sm}}^{met} = 8.91 \times 10^{-5} \]
\[ C_{ts}^{cal} = 1.86 \times 10^{-4} \]
\[ C_{p_{sm}}^{cal} = 8.0 \times 10^{-5} \]
\[ C_{p_{sm}}^{prov} = 3.72 \times 10^{-4} \]
\[ C_{p_{sm}}^{prov} = 8.8 \times 10^{-5} \]
\[ A_{steel}^{m,\Delta p,c} = 1.11375. \]

Then, since from eqn. A.n,
\[
\left( \frac{\delta (A_{\Delta p,mod})}{A_{\Delta p}^{m,\Delta p,c}} \right)^2 = \left( 1 + \frac{C_{p_{prov}}^{met}}{C_{p_{prov}}^{PLM}} - \frac{C_{p_{prov}}^{met}}{C_{p_{prov}}^{PLM clans}} \right)^2 \left( \frac{\delta (C_{p_{prov}}^{mod})}{C_{p_{prov}}^{PLM}} \right)^2 + \left( 1 + \frac{C_{p_{prov}}^{met}}{C_{p_{prov}}^{PLM}} - \frac{C_{p_{prov}}^{met}}{C_{p_{prov}}^{PLM clans}} \right)^2 \left( \frac{\delta (C_{p_{prov}}^{mod})}{C_{p_{prov}}^{PLM clans}} \right)^2.
\]

Inserting numbers presented in the example description, from equation A-n, the liquid expansion uncertainty contribution is:
\[
\left( \frac{\delta (A_{\Delta p,mod})}{A_{\Delta p}^{m,\Delta p,c}} \right)^2 = 6.142 \times 10^{-7}.
\]

From equation A-n, the steel expansion contribution is:
\[
\left( \frac{\delta (A_{\Delta p,steel,mod})}{A_{\Delta p,steel}^{m,\Delta p,c}} \right)^2 = 2.232 \times 10^{-9}.
\]

Calculating the total uncertainty contribution from the combined liquid and steel, from equation A-n:
\[
\left( \frac{\delta (A_{\Delta p,mod}^{m,\Delta p,c})}{A_{\Delta p}^{m,\Delta p,c}} \right)^2 = 6.164 \times 10^{-7}.
\]

**Calibration Uncertainty Term**

\[
\left( \frac{\delta (q_{v_o}^{cal})}{q_{v_o}^{cal}} \right)^2 = \left( \frac{\delta (q_{v_o,ref}^{cal})}{q_{v_o}^{cal}} \right)^2 + \left( \frac{\delta (q_{v_o,ref-prov}^{cal})}{q_{v_o}^{cal}} \right)^2
\]
Here the uncertainty term $q_{v_{o,prov}}^{cal}$ is in reference to the proving system at on-site calibration of proving system (i.e. water draw). The first term refers to the absolute uncertainty in the reference calibration device used to calibrate the prover. The second term is the repeatability uncertainty in checks carried out under such calibrations.

From the calibration data,

$$\left( \frac{\delta(q_{v_{o,prov}}^{cal})}{q_{v_{o,prov}}^{cal}} \right)^2 = (0.00025)^2 = 6.25 \times 10^{-8}$$

$$\left( \frac{\delta(q_{v_{o,sept-prov}}^{cal})}{q_{v_{o,prov}}^{cal}} \right)^2 = (0.00017)^2 = 2.89 \times 10^{-10}$$

$$\left( \frac{\delta(q_{v_{o}}^{cal})}{q_{v_{o}}^{cal}} \right)^2 = 6.25 \times 10^{-8} + 2.89 \times 10^{-10} = 6.28 \times 10^{-8}$$

**Prover Uncertainty Term:**

This term is uncertainty due to the proving process (not caused by calibration). This applies to single flow proving only.

$$\left( \frac{\delta(q_{v_{o,prov}}^{prov})}{q_{v_{o,prov}}^{prov}} \right)^2 = \left( \frac{\delta(q_{v_{o,prover}}^{prov})}{q_{v_{o,prov}}^{prov}} \right)^2 + \left( \frac{\delta(q_{v_{o,flowmeter}}^{prov})}{q_{v_{o,prov}}^{prov}} \right)^2 + \left( \frac{\delta(q_{v_{o,linearity}}^{prov})}{q_{v_{o,prov}}^{prov}} \right)^2 + \left( \frac{\delta(q_{v_{o,profile}}^{prov})}{q_{v_{o,prov}}^{prov}} \right)^2$$

The first term corresponds to the repeatability of the prover and the meter while proving the meter. Normally they are merged into a single term as they are difficult to distinguish individually. This combined term relates to the true uncertainty of the prover and not the repeatability itself.

$$\left( \frac{\delta(q_{v_{o,prover}}^{prov})}{q_{v_{o,prov}}^{prov}} \right)^2 = (0.00015)^2 = 2.25 \times 10^{-8}$$

The flowmeter uncertainty during proving term is the uncertainty of the indicated gross standard volume total while proving:

$$\left( \frac{\delta(q_{v_{o,flowmeter}}^{prov})}{q_{v_{o,prov}}^{prov}} \right)^2 = \left( \frac{\delta(q_{v_{o}}^{prov})}{\partial(q_{v_{o}}^{prov})} \delta(q_{v_{o}}^{prov}) \right)^2 = \left( \frac{1}{\text{number of pulses per prover pass}} \right)^2$$

$$= \left( \frac{1}{8400 \text{ pulses per BBL}} \times (20 \text{ BBL}) \right)^2 = (6 \times 10^{-6})^2 = 3.5 \times 10^{-11}$$
The linearity term, the uncertainty of the flow rate due to the adjustment of a flow meter after flow calibration can be written as:

$$
\left( \frac{\partial}{\partial \left( q^{prov}_{v_o,\text{linearity}} \right)} \right)^2 = \frac{\delta(\kappa)}{\kappa} = \frac{1}{\kappa} \frac{\partial}{\partial \kappa} \delta(\rho) = \frac{\delta(\rho) / \sqrt{3}}{100 + \rho}
$$

Where, $\rho$ is the percentage difference between flow rate from the flow meter and the reference measurement. The expression for $\delta(\rho)$ is considered to be expanded uncertainty of $\rho$ with 100% confidence level and rectangular distribution function. The standard uncertainty of $\rho$ is then found by dividing $\delta(\rho)$ by the square root of 3.

$$
\left( \frac{\partial \left( q^{prov}_{v_o,\text{linearity}} \right)}{q^{prov}_{v_o}} \right)^2 = (0.02\%)^2 = 4.0 \times 10^{-8}
$$

The profile contains the impacts from physical parameter changes like viscosity and Reynolds number. While these can be important, the impact in this example is negligible.

$$
\left( \frac{\delta \left( q^{prov}_{v_o,\text{profile}} \right)}{q^{prov}_{v_o}} \right)^2 \approx 0
$$

Therefore,

$$
\left( \frac{\delta \left( q^{prov}_{v_o} \right)}{q^{prov}_{v_o}} \right)^2 = 2.25 \times 10^{-8} + 3.5 \times 10^{-11} + 4.0 \times 10^{-8} + 0 = (0.025\%)^2 = 6.25 \times 10^{-8}
$$

**Meter Uncertainty Term**

$$
\left( \frac{\delta \left( q^{met}_{v_o} \right)}{q^{met}_{v_o}} \right)^2 = \left( \frac{\delta \left( q^{met}_{v_o,\text{flowmeter}} \right)}{q^{met}_{v_o}} \right)^2 + \left( \frac{\delta \left( q^{met}_{v_o,\text{linearity}} \right)}{q^{met}_{v_o}} \right)^2 + \left( \frac{\delta \left( q^{met}_{v_o,\text{profile}} \right)}{q^{met}_{v_o}} \right)^2 + \left( \frac{\delta \left( q^{met}_{v_o,\text{drift}} \right)}{q^{met}_{v_o}} \right)^2
$$

The flowmeter uncertainty term is based on the repeatability of the flowmeter using vendor data or proving data (MF). This is usually in terms of percent over the range of flow. From provided operational data,

$$
\left( \frac{\delta \left( q^{met}_{v_o,\text{flowmeter}} \right)}{q^{met}_{v_o}} \right)^2 = (0.037\%)^2 = 1.369 \times 10^{-7}
$$

Since factory data is given on linearity,

$$
\left( \frac{\delta \left( q^{met}_{v_o,\text{linearity}} \right)}{q^{met}_{v_o}} \right)^2 = (0.05\%)^2 = 2.5 \times 10^{-7}
$$

Profile impacts for a positive displacement meter are minimal, so that:
Drift impacts for this application are minimal because the fluid is benign and proving is done frequently, so that:
\[
\left( \frac{\delta (q_{v_0,\text{drift}})}{q_{v_0}} \right)^2 \approx 0
\]

Therefore, the combined meter uncertainty can be calculated as:
\[
\left( \frac{\delta (q_{v_0,\text{profile}})}{q_{v_0}} \right)^2 = 1.369 \times 10^{-7} + 2.5 \times 10^{-7} + 0 + 0 = (0.062\%)^2 = 3.869 \times 10^{-7}.
\]

**Sediment & Water Uncertainty Term**

\[
\left( \frac{\delta (q_{v_0,\text{sw}})}{q_{v_0}} \right)^2,
\]

And
\[
CSW = 1 - \frac{\%SW}{100%}.
\]

Since
\[
C_{v_0}^{SW} (T, P) = \frac{q_{v_0,mc} * \%SW}{100%},
\]

and the sediment and water is a function of pressure, temperature, and sample size:
\[
SW = f(P, T, l)
\]
\[
\left( \frac{\delta (q_{v_0,\text{sw}})}{q_{v_0}} \right)^2 = \left[ \frac{1}{100q_{v_0}^{SW}} \right]^2 \left[ \frac{\partial SW}{\partial T} \right]^2 + \left[ \frac{1}{100q_{v_0}^{SW}} \right]^2 \left[ \frac{\partial SW}{\partial P} \right]^2 + \left[ \frac{1}{100q_{v_0}^{SW}} \right]^2 \left[ \frac{\partial SW}{\partial l} \right]^2
\]

Additionally,
\[
\frac{\partial SW}{\partial T} = \delta SW * C_{\frac{T}{L}}^{SW} (T)
\]
\[
\frac{\partial SW}{\partial P} = \delta SW * C_{\frac{P}{L}}^{SW} (P)
\]
\[
\frac{\partial SW}{\partial l} = \delta SW
\]
Using these relations, it can be rewritten as:

\[
\left( \frac{\partial (q_{sw})}{q_{sw}} \right)^2 = \left( \frac{1}{100q_{sw}} \right)^2 \left[ (\delta SW * C_{TL}^SW (T))^2 + (\delta SW * C_{PL}^SW (P))^2 + (\delta SW)^2 \right]
\]

If we assume that sediment and water is mostly water with temperatures and pressures at the values of the flowing oil being measured,

\[
C_{TL}^SW (T) = 0.99107
\]

\[
C_{PL}^SW (P) = 1.00049
\]

\[
\delta SW = 0.02\%
\]

\[
\left( \frac{\partial (q_{sw})}{q_{sw}} \right)^2 = \left[ \frac{(0.02 + 0.99107)^2 + (0.02 + 1.00049)^2 + (0.02)^2}{(100)^2} \right] = (0.0345\%)^2 = 1.19 \times 10^{-7}
\]

**Flow Uncertainty Term**

There is no uncertainty due to flow conditions that is unaccounted for. Therefore,

\[
\left( \frac{\delta (q_{flow})}{q_{flow}} \right)^2 = 0.
\]

**Combined Uncertainty**

From 4.2.2:

\[
\delta q_{v0,me} = q_{v0,me} * \sqrt{\left( \frac{\delta (m_{liq} A_{liq})}{m_{liq} A_{liq}} \right)^2 + \left( \frac{\delta (m_{g} A_{g,spec})}{m_{g} A_{g,spec}} \right)^2 + \left( \frac{\delta (m_{calc})}{m_{calc}} \right)^2 + \left( \frac{\delta (E_{calc})}{E_{calc}} \right)^2 + \left( \frac{\delta (m_{met})}{m_{met}} \right)^2 + \left( \frac{\delta (q_{sw})}{q_{sw}} \right)^2 + \left( \frac{\delta (q_{flow})}{q_{flow}} \right)^2}
\]

Combining uncertainty from the previous sections,

\[
\left( \frac{\delta (m_{liq} A_{liq})}{m_{liq} A_{liq}} \right)^2 = (0.0785\%)^2 = 6.164 \times 10^{-7}
\]

\[
\left( \frac{\delta (m_{g} A_{g,spec})}{m_{g} A_{g,spec}} \right)^2 = (0.025\%)^2 = 6.28 \times 10^{-8}
\]

\[
\left( \frac{\delta (m_{calc})}{m_{calc}} \right)^2 = (0.025\%)^2 = 6.25 \times 10^{-8}
\]

\[
\left( \frac{\delta (E_{calc})}{E_{calc}} \right)^2 = (0.062\%)^2 = 3.869 \times 10^{-7}
\]

\[
\left( \frac{\delta (q_{sw})}{q_{sw}} \right)^2 = (0.0345\%)^2 = 1.19 \times 10^{-7}
\]
\[
\left( \frac{\delta(q_{v,0})}{q_{v,0}} \right)^2 = 0.
\]

Combining,

\[
\frac{\delta q_{v_{0,me}}}{q_{v_{0,me}}} = \pm \sqrt{(6.164 \times 10^{-7}) + (6.28 \times 10^{-8}) + (6.25 \times 10^{-8}) + (3.869 \times 10^{-7}) + (1.19 \times 10^{-7})}
\]

or

\[
\frac{\delta q_{v_{0,me}}}{q_{v_{0,me}}} = \pm 1.12 \times 10^{-3}.
\]

In percent, \( \pm 0.112\% \) uncertainty.
EXAMPLE 2: Coriolis Meter with Compact Prover

A 3 in. Coriolis meter is nominally flowing at 500 BPH, at 100 psig and 80 °F. The meter K factor is 60,000 pulses per BBL (1428.6 pulses per gallon). Operation is routinely between 450 and 550 BPH (7,500 – 9,167 Hz).

The base prover volume (BPV) of the compact prover is 20 gallons (28,571 pulses per prover pass). The compact prover is proved at 70 °F and 100 psig. ρ=30 API. Find the uncertainty of the devices.

**Meter:**

\[ T_{\text{met}} = 80 \, \text{F}, \delta T_{\text{met}} = 0.5 \, \text{F} \]

\[ P_{\text{met}} = 114.7 \, \text{psia}, \delta P_{\text{met}} = 0.5 \, \text{PSIA} \]

\[ \rho_{\text{met}} = \frac{141.5}{131.5 + API} = 0.8761 \times 999.012 \, \frac{kg}{m^3} = 875.30 \, \frac{kg}{m^3} \]

\[ \delta API = 0.1 \]
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\[ \delta \rho_{\text{met}} = \frac{\delta \text{API}_{60}}{131.5 + \text{API} \times 60} \times 875.30 \frac{\text{kg}}{\text{m}^3} \]

\[ \delta \rho_{\text{met}} = 0.54 \frac{\text{kg}}{\text{m}^3} \]

\[ SW = 0.10\%, \delta SW = 0.02\% \]

Meter steel expansion \( \gamma_{\text{met}} = 1.86 \times 10^{-5} \text{ °F}^{-1} \) at 60 °F, and \( \beta_{\text{met}} = 8.1 \times 10^{-7} \text{ psi}^{-1} \) at 14.7 psia.

**Calibration:**

\[ T_{\text{calc}} = 70 \text{ °F}, \delta T_{\text{calc}} = 0.5 \text{ °F} \]

\[ P_{\text{calc}} = 114.7 \text{ psig}, \delta P_{\text{calc}} = 0.5 \text{ PSIA} \]

\[ \rho_{\text{calc}} = 875.30 \frac{\text{kg}}{\text{m}^3}, \delta \rho_{\text{calc}} = 0.54 \frac{\text{kg}}{\text{m}^3} \]

NIST-traceable test measures used to water draw the compact prover are accurate to 0.004 gallons in 20 gallons, or 0.02% relative accuracy, and repeatability uncertainty of 0.0017%.

**Prover:**

\[ T_{\text{prov}} = 70 \text{ °F}, \delta T_{\text{prov}} = 0.5 \text{ °F} \]

\[ P_{\text{prov}} = 124.7 \text{ psig}, \delta P_{\text{prov}} = 0.5 \text{ PSIA} \]

\[ \rho_{\text{prov}} = 875.30 \frac{\text{kg}}{\text{m}^3}, \delta \rho_{\text{prov}} = 0.54 \frac{\text{kg}}{\text{m}^3} \]

Prover steel expansion \( \gamma_{\text{prov}} = 1.86 \times 10^{-5} \text{ °F}^{-1} \) at 60 °F,

From tracking data, the proving repeatability is 0.015% at the nominal flow rate of 500 BPH. Vendor gives linearity for the prover as 0.02% of full scale between 1 and 1000 BPH.

**System Uncertainty**

The terminology used for variables in this example was discussed previously in section 4. From 4.2.2 the system uncertainty can be calculated as:
\[
\Delta q_{v_o,me} = \pm \sqrt{\left( \frac{\delta (A_{liq} A_{steel})}{A_{liq} A_{steel}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2}
\]

Each term under the root is discussed and calculated below.

**Oil & Steel Expansion Uncertainty Term**

\[
\left( \frac{\delta (A_{liq} A_{steel})}{A_{liq} A_{steel}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2
\]

Where:

\[
A_{liq} = \left( \frac{c_{prov} c_{prov} c_{met} c_{met} c_{plm} c_{plm}}{c_{plm} c_{plm}} \right)
\]

\[
A_{steel} = \left( \frac{c_{prov} c_{prov} c_{prov} c_{prov} c_{prov} c_{prov}}{c_{prov} c_{prov} c_{prov}} \right)
\]

These functions are dependent upon the temperature, pressure, and density such that:

\[
A_{liq} A_{steel} = f_2(T_{cal}, P_{cal}, T_{prov}, P_{prov}, T_{met}, P_{met}, \rho_o).
\]

Given its dependencies, the oil and steel error terms can be written as:

\[
\left( \frac{\delta (A_{liq} A_{steel})}{A_{liq} A_{steel}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o})}{q_{v_o}} \right)^2
\]

If the same measurement device is used for calibration, flow measurement, and proving, that uncertainty is the same. Note that this does not have to be the case. The term “mod” implies that this part is modeled. Thus:

\[
\delta (T_p) = \delta (T_{cal}) = \delta (T_{prov})
\]
\[ \delta(P_p) = \delta(P_p^{cal}) = \delta(P_p^{prov}) \]
\[ \delta(T_m) = \delta(T_m^{prov}) = \delta(T_m^{met}) \]
\[ \delta(P_m) = \delta(P_m^{prov}) = \delta(P_m^{met}) \]

Using these relations, the error term can be rewritten as:

\[
\left( \frac{\delta(A_{liq}^{m\Delta p} A_{steel}^{m\Delta p,c})}{A_{liq}^{m\Delta p} A_{steel}^{m\Delta p,c}} \right)^2 = \frac{1}{\left[ A_{liq}^{m\Delta p} A_{steel}^{m\Delta p,c} \right]^2} \times \left[ \left( \frac{\partial f}{\partial T_p^{cal}} \right)^2 + \left( \frac{\partial f}{\partial T_p^{prov}} \right)^2 \right] \delta(T_p)^2 + \left( \frac{\partial f}{\partial P_p^{cal}} \right)^2 + \left( \frac{\partial f}{\partial P_p^{prov}} \right)^2 \delta(P_p)^2 \\
+ \left( \frac{\partial f}{\partial T_m^{prov}} \right)^2 + \left( \frac{\partial f}{\partial T_m^{met}} \right)^2 \delta(T_m)^2 + \left( \frac{\partial f}{\partial P_m^{prov}} \right)^2 + \left( \frac{\partial f}{\partial P_m^{met}} \right)^2 \delta(P_m)^2 \\
+ \left( \frac{\partial f}{\partial \rho_o} \delta(\rho_o) \right)^2 + \left( \frac{\delta(A_{liq,mod}^{m\Delta p})}{A_{liq}^{m\Delta p}} \right)^2 + \left( \frac{\delta(A_{steel,mod}^{m\Delta p,c})}{A_{steel}^{m\Delta p,c}} \right)^2 \right]
\]

The liquid portion of the term can be expanded to (eqn A.1):

\[
\left( \frac{\delta(A_{liq,mod}^{m\Delta p})}{A_{liq}^{m\Delta p}} \right)^2 = \left[ \left( \frac{\partial A_{liq}^{m\Delta p}}{\partial \beta_T} \delta(\beta_T) \right)^2 + \left( \frac{\partial A_{liq}^{m\Delta p}}{\partial \beta_P} \delta(\beta_P) \right)^2 \right] \left( A_{liq}^{m\Delta p} \right)^2 \\
= \left( 1 + \frac{C_{TLM}^{met}}{C_{TLM}^{prov}} - \frac{C_{TLM}^{met}}{C_{TLM}^{TLP}} \right)^2 \left( \delta(C_{TLM})_{mod} \right)^2 \left( \frac{C_{TLM}^{met}}{C_{TLP}^{met}} - \frac{C_{TLP}^{met}}{C_{TLP}^{TLP}} \right)^2 \left( \delta(C_{TLP})_{mod} \right)^2
\]

The general form of the expansion term due to temperature to be used above (\( C_{T_{LX}} \) from [6] 11.1.3.3 eqn. 14) can be written as:

\[ C_{T_{LX}} = e^{-\alpha \Delta T_{LX} - 0.8 a^2 \Delta T_{LX}^2} \]

where: \( \alpha = \frac{k_0}{\rho_o} + \frac{k_1}{\rho_o} + \kappa_2 \)

(note that \( \kappa_1, \kappa_2 \) are zero for crude oils; \( \kappa_0 = 341.0957 \ kg/m^6 \ ^\circ F \).

\[ \Delta T_{LX} = T_{LX} - T_o \]
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$T_0$ is the reference temperature (in this case, 60F).

The general form of the expansion term due to pressure effect ($C_{PLx}$ from [6] 11.1.3.3 eqn. 15), likewise can be written as:

$$C_{PLx} = \frac{1}{[1 - 100(P_x - P_e)F]}$$

$P_e$ is the reference or the vapor pressure of the liquid.

$$F = e^{A + \beta T_x + \rho_0^2(C + DT_x)}$$


The steel portion can also be expanded:

$$\frac{\delta (A_{steel,mod})^2}{\delta (A_{steel})^2},$$

where,

$$A_{steel,mod} = \left(\frac{C_{prov,prov,met,met}}{C_{exp,exp,exp,exp}}\right) .$$

Given

$$C_{TSx}^met = 1 + \gamma_{met}(T_x - T_b)$$ for the meter, and

$$C_{TSx}^{cal} = 1 + \gamma_{prov}(T_x - T_b)$$ for the prover calibration,

$$C_{TSx}^{prov} = 1 + \gamma_{prov}(T_x - T_b)$$ for the prover

where $T_x$ is the temperature of operation and $T_b$ is the base temperature for the expansion factors $\gamma_{met}$ and $\gamma_{prov}$. Further, for Coriolis meters and displacement provers:

$$C_{PSx}^met = 1 + \beta_{met}(P_x - P_b)$$

$$C_{PSx}^{cal} = 1 + (P_x - P_b)\left(\frac{IP}{E \times WT}\right)$$

$$C_{PSx}^{prov} = 1 + (P_x - P_b)\left(\frac{IP}{E \times WT}\right)$$

where $P_x$ is the operating pressure, $P_b$ is the base pressure for the expansion factor, $\beta_{met}$ is the expansion factor for the meter steel, $IP$ is the inner diameter of the pipe, $E$ is the Young’s modulus of the pipe, and $WT$ is the pipe wall thickness (from MPMS Chapter 12.2 [5]).

By chain differentiation:
\[
\left( \frac{\partial (A^{m, \Delta p, c})}{A^{m, \Delta p, c}} \right)^2 = \left( \frac{\partial (A^{m, \Delta p, c})}{\partial \alpha_m} \delta \alpha_m \right)^2 + \left( \frac{\partial (A^{m, \Delta p, c})}{\partial \beta_m} \delta \beta_m \right)^2
\]

With the numbers presented in the description of the example, the liquid expansion part gives:

\[
C_{\text{tip}}^{\text{prov}} = 0.99554,
\]

\[
C_{\text{tip}}^{\text{prov}} = 1.00053,
\]

\[
C_{\text{tipm}}^{\text{met}} = 0.99107,
\]

\[
C_{\text{tipm}}^{\text{met}} = 1.00049,
\]

\[
C_{\text{tipm}}^{\text{prov}} = 0.99107,
\]

\[
C_{\text{tipm}}^{\text{prov}} = 1.00049.
\]

Therefore,

\[
A_{\text{liq}}^{m, \Delta p} = 0.99607.
\]

And the steel expansion part,

\[
C_{\text{tpsp}}^{\text{prov}} = 1.86 \times 10^{-4}
\]

\[
C_{\text{tpsp}}^{\text{prov}} = 8.8 \times 10^{-5}
\]

\[
C_{\text{tpsm}}^{\text{met}} = 3.72 \times 10^{-4}
\]

\[
C_{\text{tpsm}}^{\text{met}} = 8.91 \times 10^{-5}
\]

\[
C_{\text{tpsm}}^{\text{cal}} = 1.86 \times 10^{-4}
\]

\[
C_{\text{tpsm}}^{\text{cal}} = 8.0 \times 10^{-5}
\]

\[
C_{\text{tpsm}}^{\text{prov}} = 3.72 \times 10^{-4}
\]

\[
C_{\text{tpsm}}^{\text{prov}} = 8.8 \times 10^{-5}
\]

\[
A_{\text{steel}}^{m, \Delta p,c} = 1.11375.
\]

Then, since from eqn. A.n,

\[
\left( \frac{\delta (A^{m, \Delta p, c})}{A_{\text{liq}}^{m, \Delta p}} \right)^2 = \left( 1 + \frac{c_{\text{tipm}}^{\text{met}}}{c_{\text{tipm}}^{\text{prov}}} \right)^2 \left( \frac{\delta (C_{\text{tpsp}}^{\text{met}})}{C_{\text{tpsp}}^{\text{met}}} \right)^2 + \left( 1 + \frac{c_{\text{tpsm}}^{\text{met}}}{c_{\text{tpsm}}^{\text{prov}}} \right)^2 \left( \frac{\delta (C_{\text{tpsm}}^{\text{met}})}{C_{\text{tpsm}}^{\text{met}}} \right)^2.
\]
Inserting numbers presented in the example description, from equation A-\(n\), the liquid expansion uncertainty contribution is:

\[
\left( \frac{\partial \left( \frac{m_{\Delta p}^{liq,mod}}{A_{liq}^{m,\Delta p}} \right)}{A_{liq}^{m,\Delta p}} \right)^2 = 6.142 \times 10^{-7}.
\]

From equation A-\(n\), the steel expansion contribution is:

\[
\left( \frac{\partial \left( \frac{m_{\Delta p,c}^{steel,mod}}{A_{steel}^{m,\Delta p,c}} \right)}{A_{steel}^{m,\Delta p,c}} \right)^2 = 2.232 \times 10^{-9}.
\]

Calculating the total uncertainty contribution from the combined liquid and steel, from equation A-\(n\):

\[
\left( \frac{\partial \left( \frac{m_{\Delta p}^{liq} A_{liq}^{m,\Delta p}}{A_{steel}^{m,\Delta p,c} A_{liq}^{m,\Delta p}} \right)}{A_{liq}^{m,\Delta p}} \right)^2 = 6.164 \times 10^{-7}.
\]

**Calibration Uncertainty Term**

\[
\left( \frac{\partial \left( \frac{q_{cal}^{V_o}}{q_{cal}^{V_o,ref}} \right)}{q_{cal}^{V_o}} \right)^2 = \left( \frac{\partial \left( \frac{q_{cal}^{V_o,ref}}{q_{cal}^{V_o}} \right)}{q_{cal}^{V_o,ref}} \right)^2 + \left( \frac{\partial \left( \frac{q_{cal}^{V_o,repeat-prov}}{q_{cal}^{V_o}} \right)}{q_{cal}^{V_o,repeat-prov}} \right)^2
\]

Here the uncertainty term \( q_{cal}^{V_o,prov} \) is in reference to the prover system at on-site calibration of proving system (i.e. water draw). The first term refers to the absolute uncertainty in the reference calibration device used to calibrate the prover. The second term is the repeatability uncertainty in checks carried out under such calibrations.

From the calibration data,

\[
\left( \frac{\partial \left( \frac{q_{cal}^{V_o,ref}}{q_{cal}^{V_o}} \right)}{q_{cal}^{V_o}} \right)^2 = (0.0002)^2 = 4 \times 10^{-8}
\]

\[
\left( \frac{\partial \left( \frac{q_{cal}^{V_o,repeat-prov}}{q_{cal}^{V_o}} \right)}{q_{cal}^{V_o}} \right)^2 = (0.000017)^2 = 2.89 \times 10^{-10}
\]

\[
\left( \frac{\partial \left( \frac{q_{cal}^{V_o}}{q_{cal}^{V_o}} \right)}{q_{cal}^{V_o}} \right)^2 = 4 \times 10^{-8} + 2.89 \times 10^{-10} = 4.03 \times 10^{-8}
\]

**Prover Uncertainty Term:**

This term is uncertainty due to the proving process (not caused by calibration). This applies to single flow proving only.
The first term corresponds to the repeatability of the prover and the meter while proving the meter. Normally they are merged into a single term as they are difficult to distinguish individually. This combined term relates to the true uncertainty of the prover and not the repeatability itself.

\[
\left( \frac{\delta(q_{p, prov})}{q_{p, prov}} \right)^2 = \left( \frac{\delta(q_{p, prov})}{q_{p, prov}} \right)^2 + \left( \frac{\delta(q_{v, flowmeter})}{q_{p, prov}} \right)^2 + \left( \frac{\delta(q_{v, linearity})}{q_{p, prov}} \right)^2 + \left( \frac{\delta(q_{v, profile})}{q_{p, prov}} \right)^2
\]

The flowmeter uncertainty during proving term is the uncertainty of the indicated gross standard volume total while proving:

\[
\left( \frac{\delta(q_{p, prov})}{q_{p, prov}} \right)^2 = (0.015\%)^2 = 2.25 \times 10^{-8}
\]

The linearity term, the uncertainty of the flow rate due to the adjustment of a flow meter after flow calibration can be written as:

\[
\left( \frac{\delta(q_{v, linearity})}{q_{p, prov}} \right)^2 = \frac{\delta(\kappa)}{\kappa} = \frac{1}{\kappa} \frac{\partial \kappa}{\partial \kappa} \delta(\rho) = \frac{\delta(\rho)}{\sqrt{3}}
\]

Where, \( \rho \) is the percentage difference between flow rate from the flow meter and the reference measurement. The expression for \( \delta(\rho) \) is considered to be expanded uncertainty of \( \rho \) with 100% confidence level and rectangular distribution function. The standard uncertainty of \( \rho \) is then found by dividing \( \delta(\rho) \) by the square root of 3.

\[
\left( \frac{\delta(q_{v, linearity})}{q_{p, prov}} \right)^2 = (0.02\%)^2 = 4.0 \times 10^{-8}
\]

The profile contains the impacts from physical parameter changes like viscosity and Reynolds number. While these can be important, the impact in this example is negligible.

\[
\left( \frac{\delta(q_{v, profile})}{q_{p, prov}} \right)^2 \approx 0
\]

Therefore,
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\[
\left( \frac{\delta(q^\text{prop})}{q^\text{prop}} \right)^2 = 2.25 \times 10^{-8} + 1.23 \times 10^{-9} + 4.0 \times 10^{-8} + 0 = (0.0252\%)^2 = 6.25 \times 10^{-8}
\]

**Meter Uncertainty Term**

\[
\left( \frac{\partial(q^\text{met})}{q^\text{met}} \right)^2 = \left( \frac{\delta(q^\text{met}_{\text{flowmeter}})}{q^\text{met}} \right)^2 + \left( \frac{\delta(q^\text{met}_{\text{linearity}})}{q^\text{met}} \right)^2 + \left( \frac{\delta(q^\text{met}_{\text{profile}})}{q^\text{met}} \right)^2 + \left( \frac{\delta(q^\text{met}_{\text{drift}})}{q^\text{met}} \right)^2
\]

The flowmeter random uncertainty and any uncertainty due to pressure, temperature, and fluid density influence effects on the flowmeter are usually expressed in terms of percent over the range of flow. The random uncertainty of the flowmeter of 0.05% of rate is based on the published specification from the vendor data.

\[
\left( \frac{\delta(q^\text{met}_{\text{random}})}{q^\text{met}} \right) = 0.05\% = 5 \times 10^{-4}
\]

The uncertainty due to temperature influence on the flowmeter is based on the published specification from the vendor data of 0.0005 % of meter maximum rate per °C. The maximum flow rate for this meter size is (272,000 kg/h = 1954.6 BPH), so the specification is 0.0098 BPH, or 0.00195 % of 500 BPH per °C = 0.00108 % of 500 BPH per °F.

\[
\left( \frac{\partial(q^\text{IV})}{\partial T} \delta(T) \right) = 0.00108\% \text{ per } °\text{F} \cdot 0.5 °\text{F} = 0.00054\% = 5.4 \times 10^{-6}
\]

The uncertainty due to pressure influence on the flowmeter is based on the pressure compensation of 0.0006 % of rate per psi using as input a pressure transmitter reading with a specification of 0.04 % of pressure from the vendor data. The uncertainty is 0.0006 % of rate \( \times 0.04 \% \times 114.7 \text{ psia} = 0.0000275\% \) of rate per psia.

\[
\left( \frac{\partial(q^\text{IV})}{\partial P} \delta(P) \right) = 0.0000275\% \text{ per psia} \cdot 0.5 \text{ psia} = 0.000014\% = 1.4 \times 10^{-7}
\]

Based on vendor data indicating no systematic meter error as a function of density for this meter design:

\[
\left( \frac{\partial(q^\text{IV})}{\partial \rho} \delta(\rho) \right) = 0
\]

Therefore,

\[
\left( \frac{\delta(q^\text{IV})}{q^\text{IV}} \right)^2 = \left( \frac{\partial(q^\text{IV})}{\partial T} \delta(T) \right)^2 + \left( \frac{\partial(q^\text{IV})}{\partial P} \delta(P) \right)^2 + \left( \frac{\partial(q^\text{IV})}{\partial \rho} \delta(\rho) \right)^2 = (5.4 \times 10^{-6})^2 + (1.4 \times 10^{-7})^2 + 0^2
\]

\[
\left( \frac{\delta(q^\text{IV})}{q^\text{IV}} \right) = 0.00054\% \text{ or } 5.4 \times 10^{-6}.
\]
Therefore,

\[
\left( \frac{\delta (q_{v_o,flowmeter})}{q_{v_o}} \right)^2 = \left( \frac{\delta (q_{v_o,random})}{q_{v_o}} \right)^2 + \left( \frac{\delta (q_{v_o}^I)}{q_{v_o}} \right)^2 = 0.0005^2 + 0.0000054^2
\]

\[
= (0.05\%)^2 = 2.50 \times 10^{-7}
\]

**Sediment & Water Uncertainty Term**

\[
\left( \frac{\delta (q_{v_o}^{SW})}{q_{v_o}^{SW}} \right)^2,
\]

And

\[
CSW = 1 - \frac{\%SW}{100}.
\]

Since

\[
C_{v_o}^{SW} (T, P) = \frac{q_{v_o,mc} + \%SW}{100},
\]

and the sediment and water is a function of pressure, temperature, and sample size:

\[
SW = f(P, T, l)
\]

\[
\left( \frac{\delta (q_{v_o}^{SW})}{q_{v_o}^{SW}} \right)^2 = \left[ \frac{1}{100q_{v_o}^{SW}} \left( \frac{\partial SW}{\partial T} \right)^2 \right] + \left[ \frac{1}{100q_{v_o}^{SW}} \left( \frac{\partial SW}{\partial P} \right)^2 \right] + \left[ \frac{1}{100q_{v_o}^{SW}} \left( \frac{\partial SW}{\partial l} \right)^2 \right]
\]

Additionally,

\[
\frac{\partial SW}{\partial T} = \delta SW \ast C_{TL}^{SW} (T)
\]

\[
\frac{\partial SW}{\partial P} = \delta SW \ast C_{PL}^{SW} (P)
\]

\[
\frac{\partial SW}{\partial l} = \delta SW
\]

Using these relations, it can be rewritten as:

\[
\left( \frac{\partial (q_{v_o}^{SW})}{q_{v_o}^{SW}} \right)^2 = \left( \frac{1}{100q_{v_o}^{SW}} \right)^2 \left[ (\delta SW \ast C_{TL}^{SW} (T))^2 + (\delta SW \ast C_{PL}^{SW} (P))^2 + (\delta SW)^2 \right]
\]

If we assume that sediment and water is mostly water with temperatures and pressures at the values of the flowing oil being measured,
\[ C_{TL}^{SW}(T) = 0.99107 \]
\[ C_{PL}^{SW}(P) = 1.00049 \]
\[ \delta SW = 0.02\% \]
\[ \left( \frac{\delta(q_{SW})}{q_{SW}} \right)^2 = \frac{[(0.02\times0.99107)^2 + (0.02\times1.00049)^2 + (0.02)^2]}{(100)^2} = (0.0345\%)^2 = 1.19 \times 10^{-7} \]

**Flow Uncertainty Term**

There is no uncertainty due to flow conditions that is unaccounted for. Therefore,
\[ \left( \frac{\delta(q_{flow})}{q_{flow}} \right)^2 = 0. \]

**Combined Uncertainty**

From 4.2.2:
\[
\frac{\delta q_{vo,me}}{q_{vo,me}} = \pm \sqrt{\left( \frac{\delta (A_{liq} A_{steel})}{A_{liq} A_{steel}} \right)^2 + \left( \frac{\delta (q_{cal})}{q_{cal}} \right)^2 + \left( \frac{\delta (q_{prov})}{q_{prov}} \right)^2 + \left( \frac{\delta (q_{met})}{q_{met}} \right)^2 + \left( \frac{\delta (q_{SW})}{q_{SW}} \right)^2 + \left( \frac{\delta (q_{flow})}{q_{flow}} \right)^2}
\]

Combining uncertainty from the previous sections,
\[ \left( \frac{\delta (A_{liq} A_{steel})}{A_{liq} A_{steel}} \right)^2 = (0.0785\%)^2 = 6.164 \times 10^{-7} \]
\[ \left( \frac{\delta (q_{cal})}{q_{cal}} \right)^2 = (0.02\%)^2 = 4.03 \times 10^{-8} \]
\[ \left( \frac{\delta (q_{prov})}{q_{prov}} \right)^2 = (0.0252\%)^2 = 6.35 \times 10^{-8} \]
\[ \left( \frac{\delta (q_{met})}{q_{met}} \right)^2 = (0.05\%)^2 = 2.50 \times 10^{-7} \]
\[ \left( \frac{\delta (q_{SW})}{q_{SW}} \right)^2 = (0.0345\%)^2 = 1.19 \times 10^{-7} \]
\[ \left( \frac{\delta (q_{flow})}{q_{flow}} \right)^2 = 0. \]

Combining,
\[
\frac{\delta q_{vo,me}}{q_{vo,me}} = \pm \sqrt{\left( 6.164 \times 10^{-7} \right) + \left( 4.03 \times 10^{-8} \right) + \left( 6.35 \times 10^{-8} \right) + \left( 2.50 \times 10^{-7} \right) + \left( 1.19 \times 10^{-7} \right)}
\]
or

\[
\frac{\delta d_{\text{v},\text{me}}}{q_{\text{v},\text{me}}} = \pm 1.04 \times 10^{-3} .
\]

In percent, \( \pm 0.104\% \) uncertainty.
Manual of Petroleum Measurement (MPMS)

[1] Chapter 4.2 Proving Systems—Displacement Provers

[2] Chapter 4.8 Operation of Proving Systems

[3] Chapter 5.2 Measurement of Liquid Hydrocarbons by Displacement


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